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Unique Fixed-Point Results for Soft Contractive Mapping

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Abstract: ***Most of the uncertainty problems cannot be solved by crisp sets theory, but can be explained using the concept of fuzzy sets, rough sets. The present research delt with the basic concept of soft set as well as some unique soft fixed-point theorems related to metric space. The established results satisfied the earlier results in specific condition.***

Keywords: **-Soft metric space, soft contractive mapping, Random variable.**

Mathematics Subject Classification: -47H10, 54H25.

I. INTRODUCTION & PRELIMINARIES

To define uncertainty for real world problems in 1999, Molodtsov [12] initiated a novel concept of soft sets theory. Soft systems provide a very general framework with the involvement of parameters. Since soft set theory has a rich potential. The modern fixed point theory was initiated by Banach S[1] . After the concept of Banach contraction principle there were lots of development in this theory .

Maji et al. [9-10] worked on soft set theory and presented an application of soft sets in decision making problems. Many researchers contributed towards many structure on soft set theory. [2-8]. M.Shabir and M.Naz [13] presented soft topological spaces and they investigated some properties of soft topological spaces. In these studies, the concept of soft point is expressed by different approaches. The detail about fixed point theorem for random operator and Convergence of an iteration leading to a solution of a random operator equation can be viewed in [14-17].

In the present paper some unique random soft fixed-point theorems are proved. Before proving main results, some basic definitions are written for convenience which is previously known literature.

Definition 1.1: Let X be an initial universe set and Ebe a set of parameters. A pair (F,E) is called a soft set over X if and only if X is a mapping from E into the set of all subsets of the set X, $i.e. F: E \to P(X)$, where $P(X)$ is the power set of X.

Definition 1.2: The intersection of two soft sets (F,A) and (G,B) over X is the soft set (H,C), where $C = A \cap B$ and $\forall \varepsilon \in C, H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.This is denoted by $(F,A) \cap (G,B) = (H,C)$.

Definition 1.3: The union of two soft sets (F,A) and (G,B) over X is the soft set, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & if\ \varepsilon \in A - B \\ G(\varepsilon), & if\ \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \varepsilon \in A \cap B \end{cases}$$

This relationship is denoted by $(F,A) \cup (G,B) = (H,C)$.

Definition 1.4: The soft set (F,A) over X is said to be a null soft set denoted by Φ if for all $\varepsilon \in A, F(\varepsilon) = \phi\ (null\ set)$.

Definition 1.5: A soft set (F,A) over X is said to be an absolute soft set, if for all $\varepsilon \in A, F(\varepsilon) = X$.

Definition 1.6: The complement of a soft set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c, A)$ where $F^c: A \to P(X)$ is mapping given by $F^c(\alpha) = X - F(\alpha), \forall \alpha \in A$.

Definition 1.7:Let $\Re$ be the set of real numbers and $B(\Re)$ be the collection of all nonempty bounded subsets of $\Re$ and E taken as a set of parameters. Then a mapping $F: E \to B(\Re)$ is called a soft real set. It is denoted by (F,E). If specifically,(F,E) is a singleton soft set, then identifying (F,E) with the corresponding soft element, it will be called a soft real number and denoted $\tilde{r}, \tilde{s}, \tilde{t}$ etc.

Definition 1.8: For two soft real numbers

(i) $\tilde{r} \leq \tilde{s}$, if $\tilde{r}(e) \leq \tilde{s}(e)$, for all $e \in E$.

(ii) $\tilde{r} \geq \tilde{s}$, if $\tilde{r}(e) \geq \tilde{s}(e)$, for all $e \in E$.

(iii) $\tilde{r} < \tilde{s}$, if $\tilde{r}(e) < \tilde{s}(e)$, for all $e \in E$.

(iv) $\tilde{r} > \tilde{s}$, if $\tilde{r}(e) > \tilde{s}(e)$, for all $e \in E$.

Definition 1.9: A soft set over X is said to be a soft point if there is exactly one $e \in E$, such that $P(e) = \{x\}$ for some $x \in X$ and $P(e') = \phi, \forall e' \in E \backslash \{e\}$. It will be denoted by $\tilde{x}_e$.

Definition 1.10: Two soft points $\tilde{x}_e, \tilde{y}_e$ are said to be equal if $e = e'$ and $P(e) = P(e')$i.e.$x = y$. Thus $\tilde{x}_e \neq \tilde{y}_e \Leftrightarrow x \neq y$ or $e \neq e'$.

Definition1.11: A mapping $\tilde{d}: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$, is said to be a soft metric on the soft set $\tilde{X}$ if d satisfies the following conditions:

(M1) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) \geqq \bar{0}$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \in \tilde{X}$,

(M2) $\tilde{d}(\tilde{x}_{e_1}, \tilde{y}_{e_2}) = \bar{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$,

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(M3) $\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2}) \cong \tilde{d}(\tilde{y}_{e_2},\tilde{x}_{e_1})$ for all $\tilde{x}_{e_1},\tilde{y}_{e_2} \tilde{\in} \tilde{X}$,

(M4) $\tilde{d}(\tilde{x}_{e_1},\tilde{z}_{e_3}) \cong \tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2}) + \tilde{d}(\tilde{y}_{e_2},\tilde{z}_{e_3})$ for all $\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{z}_{e_3} \tilde{\in} \tilde{X}$.

The soft set $\tilde{X}$ with a soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X},\tilde{d},E)$.

Definition 1.12 (Cauchy Sequence): A sequence $\{\tilde{x}_{\lambda,n}\}_n$ of soft points in $(\tilde{X},\tilde{d},E)$ is considered as a Cauchy sequence in $\tilde{X}$ if corresponding to every $\tilde{\varepsilon} \cong \overline{0}, \exists m \in N$ such that $d(\tilde{x}_{\lambda,i},\tilde{x}_{\lambda,j}) \cong \tilde{\varepsilon}, \forall\, i,j \geq m$, i.e. $d(\tilde{x}_{\lambda,i},\tilde{x}_{\lambda,j}) \to \overline{0}$, as $i,j \to \infty$.

Definition 1.13(Soft Complete Metric Space): A soft metric space $(\tilde{X},\tilde{d},E)$ is called complete, if every Cauchy Sequence in $\tilde{X}$ converges to some point of $\tilde{X}$.

Definition 1.14: Let $(\tilde{X},\tilde{d},E)$ be a soft metric space. A function $(f,\varphi):(\tilde{X},\tilde{d},E) \to (\tilde{X},\tilde{d},E)$ is called a soft contractive mapping if there exist a soft real number $\alpha \in R, 0 \leq \alpha < 1$ such that for every point $\tilde{x}_\lambda, \tilde{y}_\mu \in SP(\tilde{X})$ we have

$$\tilde{d}\left((f,\varphi)(\tilde{x}_\lambda),(f,\varphi)(\tilde{y}_\mu)\right) \leq \alpha \tilde{d}(\tilde{x}_\lambda,\tilde{y}_\mu)$$

II. RESULTS

Theorem 2.1: Let $(\tilde{X},\tilde{d},E)$ be a soft complete metric space. Suppose the soft mapping $(f,\varphi):(\tilde{X},\tilde{d},E) \to (\tilde{X},\tilde{d},E)$, ξ is a measerable selector and (Ω,Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω. A: $\tilde{X} \to \tilde{X}$ satisfies the condition

$$\tilde{d}\left((f,\varphi)(\xi\tilde{x}_\lambda),(f,\varphi)(\xi\tilde{y}_\mu)\right) \leq \alpha \frac{\tilde{d}\left(\xi\tilde{y}_\mu,(f,\varphi)(\xi\tilde{y}_\mu)\right)\left[1+\tilde{d}\left(\xi\tilde{x}_\lambda,(f,\varphi)(\xi\tilde{x}_\lambda)\right)\right]}{1+\tilde{d}(\xi\tilde{x}_\lambda,\xi\tilde{y}_\mu)}$$

$$+\gamma\tilde{d}(\xi\tilde{x}_\lambda,\xi\tilde{y}_\mu)+\delta\tilde{d}\left(\xi\tilde{y}_\mu,(f,\varphi)(\xi\tilde{y}_\mu)\right) \quad \textbf{(2.1.1)}$$

Where $\alpha,\beta,\gamma,\delta > 0$ and $\alpha+\gamma+\delta < 1$ is a soft constant. Then (f,φ) has a unique random soft fixed point in $\tilde{X}$.

Proof: Let $\xi\tilde{x}_\lambda^0$ be any soft point in $SP(\tilde{X})$.

Set $\xi\tilde{x}_{\lambda_1}^1 = (f,\varphi)(\xi\tilde{x}_\lambda^0) = \left(f(\xi\tilde{x}_\lambda^0)\right)_{\varphi(\lambda)}$

$$\xi\tilde{x}_{\lambda_2}^2 = (f,\varphi)(\xi\tilde{x}_{\lambda_1}^1) = \left(f^2(\xi\tilde{x}_\lambda^0)\right)_{\varphi^2(\lambda)}$$

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$$\xi\tilde{x}_{\lambda_{n+1}}^{n+1} = (f,\varphi)(\xi\tilde{x}_{\lambda_n}^n) = \left(f^{n+1}(\xi\tilde{x}_\lambda^0)\right)_{\varphi^{n+1}(\lambda)}, --$$

Now consider

$$\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) = \tilde{d}\left((f,\varphi)(\xi\tilde{x}_{\lambda_{n-1}}^{n-1}),(f,\varphi)(\xi\tilde{x}_{\lambda_n}^n)\right)$$

$$\leq \alpha \frac{\tilde{d}\left(\xi\tilde{x}_{\lambda_n}^n,(f,\varphi)(\xi\tilde{x}_{\lambda_n}^n)\right)\left[1+\tilde{d}\left(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},(f,\varphi)(\xi\tilde{x}_{\lambda_{n-1}}^{n-1})\right)\right]}{1+\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n)}$$

$$+\gamma\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n)+\delta\tilde{d}\left(\xi\tilde{x}_{\lambda_n}^n,(f,\varphi)(\xi\tilde{x}_{\lambda_n}^n)\right)$$

$$\leq \alpha \frac{\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n+1}}^{n+1})\left[1+\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n)\right]}{1+\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n)}$$

$$+\gamma\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n)+\delta\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n+1}}^{n+1})$$

$$\leq \alpha\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n+1}}^{n+1})+\gamma\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n)+\delta\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n+1}}^{n+1})$$

$$[1-(\alpha+\delta)]\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) \leq \gamma\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n)$$

$$\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) \leq \frac{\gamma}{[1-(\alpha+\delta)]}\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n)$$

$\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) \leq r\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n)$ Where $r = \frac{\gamma}{[1-(\alpha+\delta)]} < 1$

Similarly we can show that

$$\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^n) \leq r\tilde{d}(\xi\tilde{x}_{\lambda_{n-2}}^{n-2},\xi\tilde{x}_{\lambda_{n-1}}^{n-1})$$

Thus $\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) \leq r^n\tilde{d}(\xi\tilde{x}_{\lambda_0}^0,\xi\tilde{x}_{\lambda_1}^1)$

For any $n > m,\ m,n \in N$ and by triangle inequality

$$\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_m}^m) \leq \tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_{n-1}}^{n-1})+\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_{n-2}}^{n-2})+\ \dots\dots\dots+\tilde{d}(\xi\tilde{x}_{\lambda_{m+1}}^{m+1},\xi\tilde{x}_{\lambda_m}^m)$$

$$\leq \frac{r^m}{1-r}\tilde{d}(\tilde{x}_{\lambda_0}^0,\tilde{x}_{\lambda_1}^1)$$

We get $\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_m}^m) \leq \frac{r^m}{1-r}\tilde{d}(\xi\tilde{x}_{\lambda_0}^0,\xi\tilde{x}_{\lambda_1}^1)$. This implies $\tilde{d}(\xi\tilde{x}_{\lambda_n}^n,\xi\tilde{x}_{\lambda_m}^m) \to 0$ as $m,n \to \infty$.

Hence $\{\xi\tilde{x}_{\lambda_n}^n\}$ is a soft Cauchy sequence. By the completeness of $\tilde{X}$, there is $\xi\tilde{x}_\lambda^* \in \tilde{X}$ such that $\xi\tilde{x}_{\lambda_n}^n \to \xi\tilde{x}_\lambda^*,\ n \to \infty$.

Then

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$$\tilde{d}(\xi\tilde{x}_{\lambda}^{*},(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})) \leq \tilde{d}(\xi\tilde{x}_{\lambda}^{*},\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) + \tilde{d}\left(\xi\tilde{x}_{\lambda_{n+1}}^{n+1},(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})\right)$$

$$\leq \tilde{d}(\xi\tilde{x}_{\lambda}^{*},\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) + \tilde{d}\left((f,\varphi)(\xi\tilde{x}_{\lambda_n}^{n}),(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})\right)$$

$$\leq \tilde{d}(\xi\tilde{x}_{\lambda}^{*},\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) + \alpha\frac{\tilde{d}\left(\xi\tilde{x}_{\lambda}^{*},(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})\right)\left[1+\tilde{d}\left(\xi\tilde{x}_{\lambda_n}^{n},(f,\varphi)(\xi\tilde{x}_{\lambda_n}^{n})\right)\right]}{1+\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda}^{*})}$$

$$+\gamma\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda}^{*}) + \delta\tilde{d}\left(\xi\tilde{x}_{\lambda}^{*},(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})\right)$$

$$\leq \tilde{d}(\xi\tilde{x}_{\lambda}^{*},\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) + \alpha\frac{\tilde{d}\left(\xi\tilde{x}_{\lambda}^{*},(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})\right)\left[1+\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_{n+1}}^{n+1})\right]}{1+\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda}^{*})}$$

$$+\gamma\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda}^{*}) + \delta\tilde{d}\left(\xi\tilde{x}_{\lambda}^{*},(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})\right)$$

Letting $n\to\infty$ then we have

$$\tilde{d}(\xi\tilde{x}_{\lambda}^{*},(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})) \leq (\alpha+\delta)\tilde{d}\left(\xi\tilde{x}_{\lambda}^{*},(f,\varphi)(\tilde{x}\xi_{\lambda}^{*})\right)$$

But $\tilde{d}\left(\xi\tilde{x}_{\lambda}^{*},(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})\right) \geq 0$

Which gives

$$\tilde{d}(\xi\tilde{x}_{\lambda}^{*},(f,\varphi)(\xi\tilde{x}_{\lambda}^{*})) = 0 \Rightarrow (f,\varphi)(\xi\tilde{x}_{\lambda}^{*}) = \xi\tilde{x}_{\lambda}^{*}.$$

Hence $\xi\tilde{x}_{\lambda}^{*}$ is a fixed point of (f,φ).

Uniqueness: Let $\xi\tilde{y}_{\mu}^{*}$ is another fixed point of (f,φ) in $\tilde{X}$ such that $\xi\tilde{x}_{\lambda}^{*} \neq \xi\tilde{y}_{\mu}^{*}$, then we have

$$\tilde{d}(\xi\tilde{x}_{\lambda}^{*},\xi\tilde{y}_{\mu}^{*}) = \tilde{d}\left((f,\varphi)(\xi\tilde{x}_{\lambda}^{*}),(f,\varphi)(\xi\tilde{y}_{\mu}^{*})\right)$$

$$[1-\gamma]\tilde{d}(\xi\tilde{x}_{\lambda}^{*},\xi\tilde{y}_{\mu}^{*}) \leq 0$$

That is $\tilde{d}(\xi\tilde{x}_{\lambda}^{*},\xi\tilde{y}_{\mu}^{*}) = 0$ since $\gamma < 1$.

$$\Rightarrow \xi\tilde{x}_{\lambda}^{*} = \xi\tilde{y}_{\mu}^{*}$$

Hence random fixed point of (f,φ) is unique.

Theorem 2.2: Let $(\tilde{X},\tilde{d},E)$ be a soft complete space. ξ is a measerable selector and (Ω,Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω. Let mapping $(f,\varphi):(\tilde{X},\tilde{d},E)\to(\tilde{X},\tilde{d},E)$ satisfies the following soft contractive condition:

$$\tilde{d}\left((f,\varphi)(\xi\tilde{x}_{\lambda}),(f,\varphi)(\xi\tilde{y}_{\mu})\right) \leq \alpha\frac{\tilde{d}(\xi\tilde{x}_{\lambda},(f,\varphi)(\xi\tilde{x}_{\lambda})).\tilde{d}\left(\xi\tilde{x}_{\lambda},(f,\varphi)(\xi\tilde{y}_{\mu})\right)+\tilde{d}\left(\xi\tilde{y}_{\mu},(f,\varphi)(\xi\tilde{y}_{\mu})\right).\tilde{d}\left(\xi\tilde{y}_{\mu},(f,\varphi)(\xi\tilde{x}_{\lambda})\right)}{\tilde{d}\left(\tilde{x}_{\lambda},(f,\varphi)(\tilde{y}_{\mu})\right)+\tilde{d}\left(\tilde{y}_{\mu},(f,\varphi)(\tilde{x}_{\lambda})\right)}$$

$$+\beta\tilde{d}\left(\xi\tilde{y}_{\mu},(f,\varphi)(\xi\tilde{y}_{\mu})\right) + \gamma\tilde{d}(\xi\tilde{x}_{\lambda},\xi\tilde{y}_{\mu}) \quad (3.2.1)$$

Where $\alpha,\gamma > 0$ and $\alpha+\gamma < 1$ is a soft constant. Then (f,φ) has a unique random soft fixed point in $\tilde{X}$.

Proof: Let $\tilde{x}_{\lambda}^{0}$ be any soft point in $SP(\tilde{X})$.

Set $\xi\tilde{x}_{\lambda_1}^{1} = (f,\varphi)(\tilde{x}_{\lambda}^{0}) = \left(f(\xi\tilde{x}_{\lambda}^{0})\right)_{\varphi(\lambda)}$

$$\xi\tilde{x}_{\lambda_2}^{2} = (f,\varphi)(\xi\tilde{x}_{\lambda_1}^{1}) = \left(f^{2}(\xi\tilde{x}_{\lambda}^{0})\right)_{\varphi^{2}(\lambda)}$$

$$\xi\tilde{x}_{\lambda_{n+1}}^{n+1} = (f,\varphi)(\xi\tilde{x}_{\lambda_n}^{n}) = \left(f^{n+1}(\xi\tilde{x}_{\lambda}^{0})\right)_{\varphi^{n+1}(\lambda)}, --$$

Now consider

$$\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) = \tilde{d}\left((f,\varphi)(\xi\tilde{x}_{\lambda_{n-1}}^{n-1}),(f,\varphi)(\xi\tilde{x}_{\lambda_n}^{n})\right) \leq$$

$$\alpha\frac{\tilde{d}\left(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},(f,\varphi)(\xi\tilde{x}_{\lambda_{n-1}}^{n-1})\right).\tilde{d}\left(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},(f,\varphi)(\xi\tilde{x}_{\lambda_n}^{n})\right)+\tilde{d}\left(\xi\tilde{x}_{\lambda_n}^{n},(f,\varphi)(\xi\tilde{x}_{\lambda_n}^{n})\right).\tilde{d}\left(\xi\tilde{x}_{\lambda_n}^{n},(f,\varphi)(\xi\tilde{x}_{\lambda_{n-1}}^{n-1})\right)}{\tilde{d}\left(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},(f,\varphi)(\xi\tilde{x}_{\lambda_n}^{n})\right)+\tilde{d}\left(\xi\tilde{x}_{\lambda_n}^{n},(f,\varphi)(\xi\tilde{x}_{\lambda_{n-1}}^{n-1})\right)}$$

$$+\gamma\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^{n})$$

$$\leq \alpha\frac{\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^{n}).\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_{n+1}}^{n+1})+\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_{n+1}}^{n+1}).\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_n}^{n})}{\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_{n+1}}^{n+1})+\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_n}^{n})}$$

$$+\gamma\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^{n})$$

$$\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) \leq \left(\frac{\alpha+\gamma}{1}\right)\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^{n})$$

$\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) \leq s\tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_n}^{n})$ Where $s=\left(\frac{\alpha+\gamma}{1}\right) < 1$

Thus $\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_{n+1}}^{n+1}) \leq s^{n}\tilde{d}(\xi\tilde{x}_{\lambda_0}^{0},\xi\tilde{x}_{\lambda_1}^{1})$

For any $n > m,\ m,n \in N$ and by triangle inequality

$$\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_m}^{m}) \leq \tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_{n-1}}^{n-1}) + \tilde{d}(\xi\tilde{x}_{\lambda_{n-1}}^{n-1},\xi\tilde{x}_{\lambda_{n-2}}^{n-2}) + \dots\dots\dots + \tilde{d}(\xi\tilde{x}_{\lambda_{m+1}}^{m+1},\xi\tilde{x}_{\lambda_m}^{m})$$

$$\leq \frac{s^{m}}{1-s}\tilde{d}(\xi\tilde{x}_{\lambda_0}^{0},\xi\tilde{x}_{\lambda_1}^{1})$$

We get $\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_m}^{m}) \leq \frac{s^{m}}{1-s}\tilde{d}(\xi\tilde{x}_{\lambda_0}^{0},\xi\tilde{x}_{\lambda_1}^{1})$. This implies $\tilde{d}(\xi\tilde{x}_{\lambda_n}^{n},\xi\tilde{x}_{\lambda_m}^{m}) \to 0$ as $m,n\to\infty$.

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Hence $\{\xi\tilde{x}^n_{\lambda_n}\}$ is a soft Cauchy sequence. By the completeness of $\tilde{X}$, there is $\xi\tilde{x}^*_\lambda \in \tilde{X}$ such that $\xi\tilde{x}^n_{\lambda_n} \to \xi\tilde{x}^*_\lambda,\ n \to \infty$.

Then

$$\tilde{d}(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{x}^*_\lambda)) \le \tilde{d}\left(\xi\tilde{x}^*_\lambda,\xi\tilde{x}^{n+1}_{\lambda_{n+1}}\right) + \tilde{d}\left(\xi\tilde{x}^{n+1}_{\lambda_{n+1}},(f,\varphi)(\xi\tilde{x}^*_\lambda)\right)$$

$$\le \tilde{d}\left(\xi\tilde{x}^*_\lambda,\xi\tilde{x}^{n+1}_{\lambda_{n+1}}\right) + \tilde{d}\left((f,\varphi)(\xi\tilde{x}^n_{\lambda_n}),(f,\varphi)(\xi\tilde{x}^*_\lambda)\right)$$

$$\le \tilde{d}\left(\xi\tilde{x}^*_\lambda,\xi\tilde{x}^{n+1}_{\lambda_{n+1}}\right) + \alpha\frac{\tilde{d}\left(\xi\tilde{x}^n_{\lambda_n},(f,\varphi)(\xi\tilde{x}^n_{\lambda_n})\right).\tilde{d}\left(\xi\tilde{x}^n_{\lambda_n},(f,\varphi)(\xi\tilde{x}^*_\lambda)\right) + \tilde{d}\left(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{x}^*_\lambda)\right).\tilde{d}\left(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{x}^n_{\lambda_n})\right)}{\tilde{d}\left(\xi\tilde{x}^n_{\lambda_n},(f,\varphi)(\xi\tilde{x}^*_\lambda)\right) + \tilde{d}\left(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{x}^n_{\lambda_n})\right)}$$

$$+\gamma\tilde{d}\left(\xi\tilde{x}^n_{\lambda_n},\xi\tilde{x}^*_\lambda\right)$$

$$\le \tilde{d}\left(\xi\tilde{x}^*_\lambda,\xi\tilde{x}^{n+1}_{\lambda_{n+1}}\right) + \alpha\frac{\tilde{d}\left(\xi\tilde{x}^{n+1}_{\lambda_{n+1}},\xi\tilde{x}^{n+1}_{\lambda_{n+1}}\right).\tilde{d}\left(\xi\tilde{x}^n_{\lambda_n},(f,\varphi)(\xi\tilde{x}^*_\lambda)\right) + \tilde{d}\left(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{x}^*_\lambda)\right).\tilde{d}\left(\xi\tilde{x}^*_\lambda,\xi\tilde{x}^{n+1}_{\lambda_{n+1}}\right)}{\tilde{d}\left(\xi\tilde{x}^n_{\lambda_n},(f,\varphi)(\xi\tilde{x}^*_\lambda)\right) + \tilde{d}\left(\xi\tilde{x}^*_\lambda,\ \xi\tilde{x}^{n+1}_{\lambda_{n+1}}\right)}$$

$$+\gamma\tilde{d}\left(\xi\tilde{x}^n_{\lambda_n},\xi\tilde{x}^*_\lambda\right)$$

Letting $n \to \infty$ then we have

$$\tilde{d}(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{x}^*_\lambda)) \le 0$$

Which gives

$$\tilde{d}(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{x}^*_\lambda)) = 0 \Rightarrow (f,\varphi)(\xi\tilde{x}^*_\lambda) = \xi\tilde{x}^*_\lambda.$$

Hence $\xi\tilde{x}^*_\lambda$ is a fixed point of (f,φ).

Uniqueness: Let $\xi\tilde{y}^*_\mu$ is another fixed point of (f,φ) in $\tilde{X}$ such that $\xi\tilde{x}^*_\lambda \neq \xi\tilde{y}^*_\mu$, then we have

$$\tilde{d}(\xi\tilde{x}^*_\lambda,\xi\tilde{y}^*_\mu) = \tilde{d}\left((f,\varphi)(\xi\tilde{x}^*_\lambda),(f,\varphi)(\xi\tilde{y}^*_\mu)\right)$$

$$\le \alpha\frac{\tilde{d}\left(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{x}^*_\lambda)\right).\tilde{d}\left(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{y}^*_\mu)\right) + \tilde{d}\left(\xi\tilde{y}^*_\mu,(f,\varphi)(\xi\tilde{y}^*_\mu)\right).\tilde{d}\left(\xi\tilde{y}^*_\mu,(f,\varphi)(\xi\tilde{x}^*_\lambda)\right)}{\tilde{d}\left(\xi\tilde{x}^*_\lambda,(f,\varphi)(\xi\tilde{y}^*_\mu)\right) + \tilde{d}\left(\xi\tilde{y}^*_\mu,(f,\varphi)(\xi\tilde{x}^*_\lambda)\right)}$$

$$+\gamma\tilde{d}(\xi\tilde{x}^*_\lambda,\xi\tilde{y}^*_\mu)$$

$$[1-\gamma]\tilde{d}(\xi\tilde{x}^*_\lambda,\xi\tilde{y}^*_\mu) \le 0$$

That is $\tilde{d}(\xi\tilde{x}^*_\lambda,\xi\tilde{y}^*_\mu) = 0$ since $\gamma < 1$.

$$\Rightarrow \xi\tilde{x}^*_\lambda = \xi\tilde{y}^*_\mu$$

Hence fixed point of (f,φ) is unique.

Theorem 2.3: Let $(\tilde{X},\tilde{d},E)$ be a soft complete space. ξ is a measerable selector and (Ω,Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω. Let mapping $(f,\varphi):(\tilde{X},\tilde{d},E) \to (\tilde{X},\tilde{d},E)$ satisfies the following soft contractive condition:

$$\tilde{d}\left((f,\varphi)(\xi\tilde{x}_\lambda),(f,\varphi)(\xi\tilde{y}_\mu)\right) \le \alpha\tilde{d}\left(\xi\tilde{x}_\lambda,\tilde{y}\xi_\mu\right)$$

$$+\gamma\frac{\tilde{d}\left(\xi\tilde{y}_\mu,(f,\varphi)(\xi\tilde{y}_\mu)\right)\left[1+\tilde{d}\left(\xi\tilde{x}_\lambda,(f,\varphi)(\xi\tilde{y}_\mu)\right)+\tilde{d}\left(\xi\tilde{y}_\mu,(f,\varphi)(\xi\tilde{x}_\lambda)\right)\right]}{1+\tilde{d}(\xi\tilde{x}_\lambda,\xi\tilde{y}_\mu)+\tilde{d}\left(\xi\tilde{y}_\mu,(f,\varphi)(\xi\tilde{y}_\mu)\right)} \quad (3.3.1)$$

Where $\alpha,\gamma > 0$ and $\alpha+\gamma < 1$ is a soft constant. Then (f,φ) has a unique random soft fixed point in $\tilde{X}$.

Proof: It can be proved as previous results.

Theorem 2.4: Let $(\tilde{X},\tilde{d},E)$ be a soft complete space. ξ is a measerable selector and (Ω,Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω. Let mapping $(f,\varphi):(\tilde{X},\tilde{d},E) \to (\tilde{X},\tilde{d},E)$ satisfies the following soft contractive condition:

$$\tilde{d}\left((f,\varphi)^{p+1}(\xi\tilde{x}_\lambda),(f,\varphi)^{p+2}(\xi\tilde{y}_\mu)\right) \le \alpha\frac{\tilde{d}\left(\xi\tilde{y}_\mu,(f,\varphi)^{p+2}(\xi\tilde{y}_\mu)\right)\left[1+\tilde{d}\left(\xi\tilde{x}_\lambda,(f,\varphi)^{p+1}(\xi\tilde{x}_\lambda)\right)\right]}{1+\tilde{d}\left(\xi\tilde{y}_\mu,(f,\varphi)^{p+1}(\xi\tilde{x}_\lambda)\right)}$$

$$+\gamma\tilde{d}\left(\xi\tilde{y}_\mu,(f,\varphi)^{p+2}(\xi\tilde{y}_\mu)\right)+\delta\tilde{d}\left(\xi\tilde{x}_\lambda,\xi\tilde{y}_\mu\right) \quad (3.4.1)$$

Where $\alpha,\gamma,\delta > 0$ and $\alpha+\gamma+\delta < \frac{1}{2}$ is a soft constant and for any non-negative integer p. Then (f,φ) has a unique random soft fixed point in $\tilde{X}$.

Proof: It can be proved easily taking particular value of p in above results

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